The Functionally Near-Equivalency of Reynolds and Grashof Numbers

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A complete analytical solution of many problems in chemical engineering is frequently unobtainable, but it is often possible to make a problem more intelligible by judiciously combining available analytical and physical information. Thus with the stressing of the formal equivalency of Reynolds and Grashof numbers, forced and natural heat transfer equations become closely interrelated, thereby increasing the utility of the equations and clarifying the role of the dimensionless numbers.

COMPARISON OF FORCED AND NATURAL CONVECTION SYSTEMS

Two geometrically similar heat transfer systems with incompressible forced convection flow over exterior surfaces will be in states of dynamic similarity if the Reynolds numbers are equal (Kreith, 1973a).

Analogously, two geometrically similar heat transfer surfaces with incompressible, natural convection flow over exterior surfaces will be in states of dynamic similarity if the Grashof numbers are equal (Kreith, 1973b).

Logically it follows from these theoretical and experimentally confirmed facts that the Reynolds and Grashof numbers play equivalent roles in forced and natural convection systems, respectively.

This equivalency is also evident from the conventional definitions of the two numbers, $Re = LV/\nu$ and $Gr = g(\beta\Delta t)L^3/\nu^2$ or writing a modified Grashof number in the form $Gr' = L[g(\beta\Delta t)L]^{1/2}/\nu^2$ to clarify this. Such a redefinition of a Grashof number $Gr' = Gr^{1/2}$ is not at all unusual; many dimensionless numbers have different formal definitions appropriate and convenient for solving various problems (Olson, 1973).

Closer examination of the quantity $[g(\beta\Delta t)L]^{1/2}$ shows that it indeed has the dimensions of a velocity. It is the velocity which will be attained by a fluid particle rising or falling without frictional resistance. When at the film temperature and surrounded by fluid at the ambient temperature, the distance through which the fluid particle is accelerated is the characteristic length L and the acceleration of the gravitational field is g. The buoyancy force will of course also depend on the dimensionless number $\beta\Delta t$ as indicated.

AGREEMENT WITH EXPERIMENTAL DATA

To test the equivalency of the Reynolds number and the modified Grashof number in a particular case, the similarity of the temperature fields near the surface of a horizontal cylinder is compared by plotting the Nusselt number against $Re_f Pr_f^{1/2}$ and $Gr_f' Pr_f^{1/2}$ for two sets of published data [for forced convection the data published by Hilpert (1933) and for natural convection the data published by McAdams (1954)]. All dimensionless numbers were evaluated at the film temperature. The abscissa of the first data set has been shifted to correspond to multiplication of the Reynolds number by $Pr^{1/2} = 0.74^{1/2} = 0.86$, and for the second set the square root of the Grashof number multiplied by the Prandtl number has been extracted before plotting. The results are shown in Figure 1.

The predicted performance, based on modification of the Grashof number, agrees with the experimental measurements with an accuracy typical of convective heat transfer experiments. For $Re_f Pr_f^{1/2}$ ranging from 3 to 10^4 the error is less than 10%. This is very surprising since the velocity fields are essentially different, the forces maintaining them differ, and the forced convection field acts at the control volume boundaries while the natural convective field is maintained by body forces acting within the control volume.

The discrepancies between the two curves, at very low and very high Reynolds or Grashof numbers, may perhaps be ascribed to the experimental conditions under which the data have been obtained. Forced convection data become somewhat dubious at low Reynolds numbers because the interference of natural convection. High Grashof numbers produced by large temperature differences vitiate the assumption that Grashof and Nusselt numbers may be correlated at the defined film temperature $t_f = \frac{1}{2}(t_s + t_x)$.

From the stated viewpoint, the Grashof number can be considered a nonstandard Reynolds number; alternatively it has been represented as the product of a Reynolds number divided by a modified Froude number $V/[Lg(\beta\Delta 5)]^{1/2}$. The choice of viewpoint is simply a matter of utility.

It has previously been pointed out by Kreith (1973c) that the dimensionless group Gr/Re^2 gives a qualitative indication of the influence of buoyancy on forced convection, and the grouping Gr/Re^2 is also mentioned by White (1974).

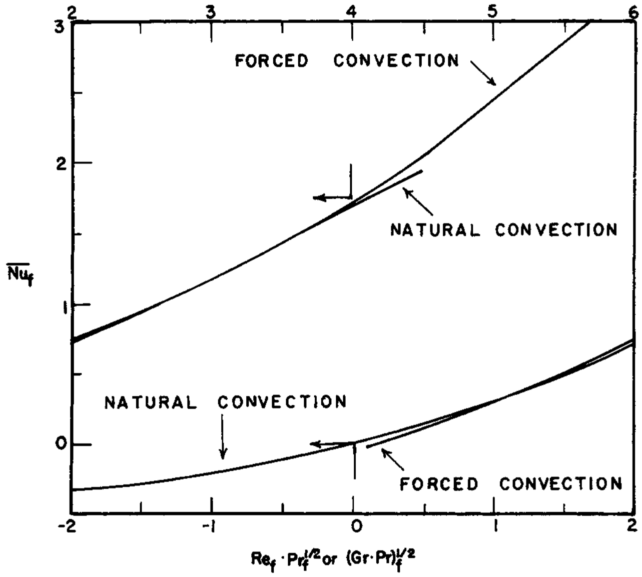

Fig. 1. Comparison of forced and natural convection correlations.

QUESTIONS TO BE ANSWERED

Some interesting problems arise; will the drag coefficients also coincide for forced and natural convection when plotted against Reynolds or modified Grashof numbers just as the Nusselt numbers do?

Should the velocity $[g(\beta\Delta t)L]^{1/2}$ be added vectorially to the velocity V to arrive at an appropriate velocity for use in a Reynolds number that will predict the heat transfer coefficient with mixed forced and natural convection flow?

NOTATION

c_p = heat capacity of fluid, $Wkg^{-1}K^{-1}$
Fr = Froude number, $V/(gL)^{1/2}$ dimensionless
g = acceleration of gravity, ms^{-2}
Gr = Grashof number, dimensionless
Gr' = modified Grashof number $Gr^{1/2}$, dimensionless
$\bar{h}$ = circumferentially averaged surface coefficient of heat transfer, $Wm^{-2}K^{-1}$
k = thermal conductivity of fluid, $Wm^{-1}K^{-1}$
L = characteristic length, m
$\overline{Nu}$ = average Nusselt number, $\bar{h}L/k$, dimensionless
Pr = Prandtl number, $c_p\mu/k$, dimensionless
Re = Reynolds number, LV/ν, dimensionless
t = temperature, K
Δt = temperature difference, $t_s - t_\infty$, K
V = velocity, ms^{-1}

Greek Letters

β = temperature coefficient of thermal expansion, K^{-1}
ν = kinematic viscosity, m^2s^{-1}
μ = dynamic viscosity, $kgm^{-1}s^{-1}$

Subscripts

f = evaluated at the film temperature $\frac{1}{2}(t_s + t_\infty)$
s = at surface
∞ = of unheated fluid

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Drag Reduction in Film Flow

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It is well known that some dilute polymer solutions in water or other solvents exhibit a reduction of skin friction in turbulent flow. The most common cause mentioned in literature is in terms of extensional flow properties of dilute polymer solutions (Seyer and Metzner, 1969). The resistance of dilute polymer solutions to extensional deformation, commonly termed *elongational viscosity*, is in orders of magnitude higher than that of pure Newtonian solvents (Metzner, 1971). The structure of the flow field in turbulent pipe flow in the wall region consists of counterrotating pairs of eddies. This flow pattern consists, at least in part, of an elongational motion (Gordon et al., 1973).

Now, consider the flow pattern of a wavy film flow. There is an intensive backmixing in the wave trough, qualitatively described first by Kapitza (1948) and mathematically by Massot et al. (1966), as is shown in Figure 1. It is also clear that stretching components of deformation are present when waves appear on the film surface. Although the film flow with its free surface substantially differs from a turbulent pipe flow, there are some similarities in the flow pattern of a film and flow in the wall region of turbulent pipe flow, in particular, in the elements of stretching motion.

The observations stated above form the basis for an explanation of the remarkable reduction in friction coefficient found in experiments on falling films of dilute aqueous Carbopol 934 solutions (Popadić, 1974). For the lowest concentration examined (0.05% w), the friction coefficient was more than 50% lower than predicted by both Equations (2) and (4), as is seen in Figure 2.

Equation (2) is obtained for the case of a nonwavy laminar film flow for which

$$f = \frac{2g\delta_0^3}{Q^2} \quad (1)$$

where δ_0 is the film thickness (average thickness for the case of wavy flow) and Q is the volume flow rate per unit width of the plate, or

$$f = \frac{24}{Re} \quad (2)$$

where Re is defined as (Skelland, 1967)

$$Re = \frac{12n}{2n+1}\left(\frac{Q}{g\delta_0}\right)\left(\frac{\rho g\delta_0}{K}\right)^{1/n} \quad (3)$$

In (3) K is the consistency factor and n is the flow behavior index.

For a wavy film flow, an approximative steady periodic solution of a boundary layer form of equation of motion for a power-law liquid has led to (Popadić, 1974)

$$f = 0.833^n \cdot \frac{24}{Re}\,\Phi(n) \quad (4)$$

where $\Phi(n)$ is a determined function of the flow behavior index.

For Newtonian liquids $n = 1$ and $\Phi(n) = 1$, and Equation (4) gives smaller friction coefficient for a wavy flow than for the nonwavy laminar flow.

A very good agreement between predicted and experi-